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INTRINSIC SOURCES OF IM GENERATION.(U)

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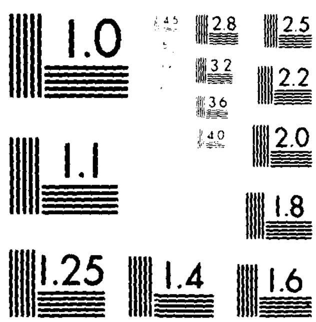

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NRL Memorandum Report 4233, CHAP 5	2. GOVT ACCESSION NO. AD-A088771	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle)		5. TYPE OF REPORT & PERIOD COVERED FINAL REPORT
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s)		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Research Laboratory Washington, D.C. 20375		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NRL Problem R08-73 Program Element 33109N Project X-0731-CC
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Electronic Systems Command Washington, D.C. 20360		12. REPORT DATE July 7, 1980
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

DTIC ELECTE SEP 5, 1980 C

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Satellite communications	Connector design
Intermodulation interference	Ferromagnetic materials
Nonlinear conduction	Nonlinear circuit analysis
Tunneling junctions	Multiplex systems

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

80 7 22 025

(14) NRL-MR-4[illegible]-5) (9) Final rept.,
(16) [illegible] (11) [illegible] (12) 19
(17) [illegible]

(6) INTRINSIC SOURCES OF IM GENERATION.

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In addition to avoidable sources of intermodulation signals introduced in manufacture or assembly of a multiplex system, there will be some sources inherent to the materials of which the system is constructed. At some point these sources must provide a lower bound to possible system sensitivity. Such a limitation will only be significant if it is reached before thermal noise becomes dominant. ~~The~~ thermal noise power is $P_N = kT\Delta\nu$ W for ~~a~~ frequency ~~bandwidth $\Delta\nu$~~. At 20°C and a bandwidth of 2500 Hz, the noise power becomes -140 dBm. It is potentially possible to lower either the temperature or the bandwidth by a factor of about 100, so an ultimate sensitivity limit of about -180 dBm is perhaps significant. Above these levels, a number of intrinsic IM mechanisms can be identified. This chapter constitutes a review of significant intrinsic IM sources.

1. RESISTIVE HEATING IN NON-MAGNETIC CONDUCTORS

This problem has been discussed in several places including Philco[1] and TRW[2] studies. This section is aimed at unifying the discussions and eliminating apparent discrepancies in the conclusions. It is concluded that this effect can be significant at high power levels at all frequencies.

The calculation which follows is done classically (no quantum effects; cf. Chapter I) and to first order. It is similar to that of the Philco study, but with fewer approximations. The TRW calculations, done differently and specifically for a waveguide configuration, reach substantially the same conclusions. Two primary frequencies ω_1 and ω_2 ($\omega_1 > \omega_2$) are assumed present with electric fields of amplitudes E_1 and $E_2 = \beta E_1$ parallel to the metallic surface, and the normally strongest 3rd order IM signal at $(2\omega_1 - \omega_2)$ is sought. The surface is taken as part of an infinite plane and $\omega_1, \omega_2, (2\omega_1 - \omega_2)$ are assumed close enough in frequency that a single skin depth $\delta_0 \equiv \left(\frac{2\rho_o}{\omega\mu}\right)^{1/2}$ in terms of symbols in the appended list can be used for all.

To lowest order, the density of primary power being dissipated at a depth z below the surface is then

$$p_{IN}(z) = \frac{E_1^2}{2\rho_0} e^{-\frac{2z}{\delta_0}} \left\{ (1+\beta^2) + \cos\left[2\omega_1 t - \frac{2z}{\delta_0}\right] + \beta^2 \cos\left[2\omega_2 t - \frac{2z}{\delta_o}\right] \right. \tag{1}$$

$$\left. + 2\beta \cos\left[(\omega_1 + \omega_2)t - \frac{2z}{\delta_0}\right] + 2\beta \cos(\omega_1 - \omega_2)t \right\},$$

where ρ_0 is the material resistivity at ambient temperature. [For copper $\rho_0 = 1.72 \times 10^{-8}\ \Omega$ m at room temperature]. This power dissipation produces resistive heating and changes the local value of the resistivity. Heating can in principle also lead to local dimensional changes which could produce intermodulation. However, with temperature changes expected to be $\leqslant 10^{-6}$K even at the surface, a linear

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thermal coefficient of 10^{-5}/K, weak dependence of the propagated signal on small dimensional changes, and considerable bulk thermal inertia, this effect is considered negligible. From (1), one finds that the total power absorbed at all depths per unit surface area, averaged in time, is

$$\bar{p}_{IN} = \frac{\delta_0 E_1^2 (1 + \beta^2)}{4\rho_0}. \tag{2}$$

To find the change in local resistivity, one uses the fact that ρ is essentially proportional to absolute temperature and then looks for the temperature distribution. This is obtained from the diffusion equation

$$-G \frac{\partial^2 T(z)}{\partial z^2} + C_h \frac{\partial T(z)}{\partial t} = p_{IN}(z). \tag{3}$$

Here G is the thermal conductivity and C_h the heat capacity. [For copper at room temperature, $G = 4.2 \times 10^2$ W/Km, $C_h = 3.44 \times 10^6$J/Km^3]. Since Eq. (3) is linear in $T(z)$, the solution will be a superposition of solutions for the various terms in $p(z)$ of Eq. (1). We neglect the steady state term; for the others, boundary conditions are taken to be $\frac{\partial T(z)}{\partial z} = 0$ at $z = 0$ (no heat flow into air space) and $T(z)_{z\to\infty} = 0$ (the thickness of ordinary components is sufficient to be considered infinite).

In solving for $T(z)$, one will obtain contributions corresponding to all the different frequency dependent power terms. When these are converted to changes in resistivity and combined with the input signals, they will yield currents at several frequencies. However, the desired $(2\omega_1 - \omega_2)$ current only arises either from modulation of the ω_1 input by an $(\omega_1 - \omega_2)$ resistivity term, or from modulation of the ω_2 input by a $2\omega_1$ term. Furthermore, evaluation of the current amplitudes shows that modulation of ω_1 by the $(\omega_1 - \omega_2)$ resistivity term is dominant. Hence, only this part will be discussed in what follows. From the solution for $T(z)$ we then obtain

$$\rho(z) = \rho_0\left(1 + \frac{T(z)}{T_0}\right) = \rho_0 + \frac{\rho_0}{T_0}\frac{\left(\frac{\beta E_1^2}{2\rho_0 G}\right)}{\left(\frac{4}{\delta_0^4} + W^4\right)^{1/2}}\left\{\frac{\sqrt{2}\, e^{-Wz}}{\delta_0 W}\cos\left[(\omega_1 - \omega_2)t - Wz + \phi - \frac{\pi}{4}\right]\right.$$
$$\left. - e^{-\frac{2z}{\delta_0}} \cos\left((\omega_1 - \omega_2)t + \phi\right)\right\}. \tag{4}$$

Here $W^2 \equiv \frac{(\omega_1 - \omega_2)C_h}{2G}$ and $\tan\phi \equiv \frac{W^2\delta_0^2}{2}$.

This resistivity will modulate the local currents produced by the primary input at ω_1. Writing the skin effect in differential form, the current density at z, $J(z)$, is found from

$$\frac{1}{J(z)}\frac{dJ(z)}{d(z)} = -\frac{(1+i)}{\delta(z)} = -\frac{(1+i)}{\delta_0\left(\frac{\rho}{\rho_0}\right)^{1/2}} \approx \frac{(1+i)}{\delta_0}\left(\frac{1 - T(z)}{2T_0}\right). \tag{5}$$

For an input at ω_1 one obtains

$$J(z) = \frac{E_1}{\rho_0} e^{-\frac{z}{\delta_0}}\left\{\left(1 - \frac{T(0)}{T_0}\right)\cos\left(\omega_1 t - \frac{z}{\delta_0}\right) + \frac{\int_0^z T(z)\,dz}{\sqrt{2}\,\delta_0 T_0}\cos\left(\omega_1 t - \frac{z}{\delta_0} + \frac{\pi}{4}\right)\right\}. \tag{6}$$

When the quantities $T(0)$ and $\int_0^z T(z)dz$ are evaluated and substituted into (6), the IM current density with frequency $2\omega_1 - \omega_2$ is found to be

$$
\begin{aligned}
J_{IM}(z) = {} & \frac{\beta E_1^2}{\rho_0} \frac{E_1 \delta_0^2}{\rho_0} \frac{1}{2GT_0} (4 + x^4)^{-1/2} e^{-\frac{z}{\delta_0}} \times \\
& \left\{ \left(\frac{3}{8} - \frac{1}{2x} + \frac{1}{4x^2} \right) \cos \left[(2\omega_1 - \omega_2)t - \frac{z}{\delta_0} + \phi \right] + \left(\frac{1}{8} - \frac{1}{2x} + \frac{1}{4x^2} \right) \times \right. \\
& \sin \left[(2\omega_1 - \omega_2)t - \frac{z}{\delta_0} + \phi \right] \\
& - \frac{e^{-Wz}}{2\sqrt{2}\, x^2} \sin \left[(2\omega_1 - \omega_2)t - \frac{z}{\delta_0} + \phi + \frac{\pi}{4} - Wz \right] + \frac{e^{\frac{2z}{\delta_0}}}{4\sqrt{2}} \times \\
& \left. \cos \left[(2\omega_1 - \omega_2)t - \frac{z}{\delta_0} + \phi + \frac{\pi}{4} \right] \right\}.
\end{aligned} \tag{7}
$$

The dimensionless parameter $x \equiv \delta_0 W$ is of order of magnitude unity here, but is material dependent.

In treating an unperturbed input signal, one finds that $\bar{p}_{IN}$, the power absorbed per unit surface area, is related to the total current density in the same element, $J \equiv \int_0^\infty J(z)dz$, by way of the surface resistance $R_s \equiv \frac{\rho_0}{\delta_0}$; i.e.,

$$\bar{p}_{IN} = \overline{J^2} R_s \tag{8}$$

We will apply this same expression to find the power lost by $J_{IM} \equiv \int_0^\infty J_{IM}(z)dz$ in the same volume. Integrating Eq. (7), we obtain

$$
\begin{aligned}
J_{IM} = {} & \frac{\frac{\beta E_1^2}{\rho_0} \left(\frac{E_1 \delta_0^2}{\rho_0} \right) \delta_0}{2GT_0 (4 + x^4)^{1/2}} \left\{ \frac{7}{40} \cos \left[(2\omega_1 - \omega_2)t + \phi \right] + \left(\frac{9}{40} - \frac{1 + 2x}{4x(1 + x)} \right) \times \right. \\
& \left. \sin \left[(2\omega_1 - \omega_2)t + \phi \right] \right\}.
\end{aligned} \tag{9}
$$

Hence from (8) and (9) the total IM power absorbed per unit area becomes

$$\bar{p}_{IM} = \frac{\rho_0}{\delta_0} \overline{J_{IM}^2} = \frac{\beta^2 \left(\frac{\delta_0 E_1^2}{4\rho_0} \right)^3 \delta_0^2}{G^2 T_0^2 (4 + x^4)} \left[\frac{13}{20} - \frac{(9x^2 - x - 5)(2x + 1)}{10x^2(x + 1)^2} \right], \tag{10}$$

or, using Eq. (2),

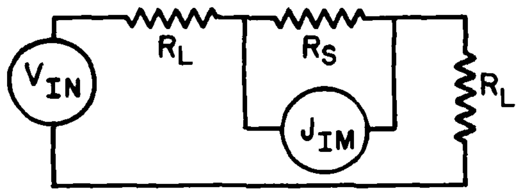
$$\bar{p}_{IM} = \frac{\beta^2}{(1 + \beta^2)^3} \frac{\bar{p}_{IN}^3 \delta_0^2}{G^2 T_0^2} f(x), \tag{11}$$

where $$f(x) \equiv \frac{1}{(4 + x^4)} \left[\frac{13}{20} - \frac{(9x^2 - x - 5)(2x + 1)}{10x^2(x + 1)^2} \right].$$

Reasonable values for x in our approximation that $(\omega_1 - \omega_2) << \omega_1$ are in the range $x = 1$ to 5 for copper. In order to satisfy the assumption of first order perturbation, $\frac{T(0)}{T_0}$ and $\int_0^z \frac{T(z)dz}{\delta_0 T_0}$ in Eq. (6) must be much less than 1. This requires that $x > \left|\frac{\delta_0 \bar{p}_0}{GT_0}\right|$, which here roughly means $x > 10^{-3}$. Hence, the infinity in $f(x)$ at $x = 0$ is meaningless. For reasonable x values, $f(x)$ is controlled mainly by the $\frac{1}{4 + x^4}$ factor. Explicit values are

x	=	1	2	3	4	5
$f(x)$	=	.085	.012	.0036	.0013	.0006

The reason for obtaining Eq. (11) is that the IM power received at the output load can be expressed in terms of this quantity. We assume that we are dealing with a well designed circuit with matched input and load. Then we can represent a unit IM source included in a linear system by the equivalent circuit illustrated.



From this one IM source the power received at the load $\bar{p}_{IML} = \overline{J_{IM}^2}\, R_S \left[\frac{R_L R_S}{(2R_L + R_S)^2}\right]$. But the input power lost in the unit IM source is $\frac{V_{IN}^2 R_S}{2(R_L + R_S)^2}$ while that available to the load is $\frac{\overline{V_{IN}^2}}{4R_L}$; the ratio of these gives $\frac{4R_S R_L}{(2R_L + R_S)^2}$. Hence we can write, using (10) or (8) and (2)

$$\bar{p}_{IML} = \frac{\bar{p}_{IN}\bar{p}_{IM}}{4P_{IN}} \tag{12}$$

for the delivered IM power from a unit IM source element. If the assumption is made that all IM signals arrive in phase at the load, the total $P_{IML} = (\text{Area})^2 \bar{p}_{IML}$. This then gives the result that

$$P_{IML} = \frac{\bar{p}_{IM}(P_{IN\,\text{lost}})^2}{4\bar{p}_{IN} P_{IN}}. \tag{13}$$

Using Eq. (11), we obtain finally

$$P_{IML} = \frac{\beta^2}{(1+\beta^2)^3}\left|\frac{\delta_0}{2GT_0}\right|^2 \bar{p}_{IN}^2 \frac{(P_{IN\,\text{lost}})^2 f(x)}{P_{IN}} \tag{14}$$

As numerical examples, we consider two cases. (i) A high-Q UHF structure with high power density in a very restricted area with $\nu_1 = 270$ MHz, $\nu_2 = 245$ MHz, $\beta = 1$, $\bar{p}_{IN} = 4.5 \times 10^3$ W/m^2, $P_{IN} = 100$ W, $P_{IN\,lost} = 20$ W, $x = 3.2$, $f(x) = .0028$. This leads to $P_{IML} = .71 \times 10^{-17}$ W or -142 dBm. (ii) The x-band waveguide case considered in the Philco report, with $\nu_1 = 8050$ MHz, $\nu_2 = 7900$ MHz, $\beta = 0.1$, $\bar{p}_{IN} = 145$ W/m^2, $P_{IN} = 1000$ W, $P_{IN\,\text{lost}} = 45$ W, $x = 1.45$, $f(x) = .031$. This gives $P_{IML} = 1.1 \times 10^{-22}$ W or -190 dBm. Correction of a numerical error brings the Philco value to -232 dBm; the remaining difference is associated with the assumption in that report that modulation of the

input at ω_2 by the thermal effects at $2\omega_1$ is dominant, whereas we find modulation of the input at ω_1 by $(\omega_1 - \omega_2)$ thermal terms produces an effect about 40 dB stronger. Both of these examples indicate that this mechanism is capable, under certain conditions, of producing significant IM signals.

A particularly important non-intrinsic IM source which has been identified is the pressure-closed junction. Because of oxidation and surface roughness this junction contains insulating, semiconducting, and metallic regions in dimensions small enough to permit tunneling. However, consideration of the restricted area of metallic contact within the skin layer and across a gap of perhaps 100 Å suggests that the junction would be an important IM source simply through resistive heating. For example, in the waveguide discussed above, if a flange junction were introduced in which 1 W is dissipated or $\bar{p}_{IN} = 2 \times 10^9$ W/m^2, then P_{IML} would be increased 110 dB to a significant level of -122 dBm. It is to be expected that increasing contact pressure would decrease this contribution to junction IM production.

To see the important dependences more clearly, we will write $P_{IN\,\text{lost}} = \bar{p}_{IN} A$ where A is the effective conductor area; $\frac{1}{T_0}$ is generalized to ψ in the expression $\rho = \rho_0(1 + \psi T)$; $\bar{p}_{IN} \propto QP_{IN}R_s = QP_{IN}\rho_0/\delta_0$, where Q is descriptive of the element in question and R_s is the surface resistivity of the conductor; $f(x) \simeq x^{-3}$ quite well in the range $1 < x < 5$ and $x = \left[\frac{\delta_0^2(\omega_1 - \omega_2)C_h}{2G}\right]^{1/2}$. Thus Eq. (14) lead to

$$
\begin{aligned}
P_{IML} &\propto \frac{\beta^2}{(1+\beta^2)^3}\left(\frac{A\psi}{2G}\right)^2\left[\frac{2G}{\delta_0^2(\omega_1 - \omega_2)C_h}\right]^{3/2}\delta_0^2 P_{IN}^3\left(\frac{\rho_0}{\delta_0}\right)^4 Q^4 \\
&\propto \frac{\beta^2}{(1+\beta^2)^3}\,\frac{A^2\psi^2Q^4}{G^{1/2}C_h^{3/2}}P_{IN}^3\rho_0^4\delta_0^{-5}(\omega_1 - \omega_2)^{-3/2} \qquad (15)\\
&\propto \frac{\beta^2}{(1+\beta^2)^3}\,\frac{A^2\psi^2Q^4}{G^{1/2}C_h^{3/2}}P_{IN}^3\rho_0^{3/2}\,\frac{\omega_1^{5/2}}{(\omega_1 - \omega_2)^{3/2}}
\end{aligned}
$$

Thus IM production due to resistive heating is enhanced by:

(a) Large power inputs,

(b) β values near 1 (equal power in both carriers),

(c) Large-area elements,

(d) High-Q elements,

(e) High carrier frequencies,

(f) Small carrier separations,

(g) Conductors with high resistivity, low heat capacity, low thermal conductivity, and high-thermal coefficient of resitivity ψ.

(It might also be noted that Eq. (14) is, to the level of approximations made, in functional and quantitative agreement with the calculations by TRW, after correction of a numerical error. The TRW result, obtained in the manner of Section VI (dielectrics), develops on the fact that for long signal paths and sufficient attenuation the value of P_{IML} will saturate and then decline for longer paths. For most applications the critical length, where the primary signals are attenuated 4.8 dB, will be quite long.)

2. MAGNETORESISTIVE GENERATION IN NON-MAGNETIC CONDUCTORS

Magnetic fields applied to a conductor have the effect of altering its resistivity. Hence those fields associated with currents in the material can potentially create IM signals. An approach similar to that used in Part 1. can be taken. We will evaluate the variations produced in the local resistivity by the primary currents and then find J_{IM}. This effect is found to be small.

First consider a simple numerical result. With $\delta_0 = 10^{-6}$m typical of copper, a dissipation $\bar{p}_0 = 10^3$ W/m^2 using (8) leads to $J \approx 240$ A rms as the current through a 1 m width of skin layer. But in MKS units we can write $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$ where I is the current through the loop around which $\vec{B}$ is integrated. Taking the loop as a rectangle transverse to the current, 1 m wide and several δ deep, with its upper side at the surface of the conductor, the contributions of the short ends to the integral cancel, and B on the lower long side is essentially zero. Hence B on the upper surface becomes roughly $B = 240\mu_0$ Tesla, which is equivalent to about 3 Gauss. But for the transverse magnetoresistive effect (B perpendicular to current flow as it will be for self-produced fields) we know[3] for copper that $\frac{\rho - \rho_0}{\rho_0} = 3.98 \times 10^{-17} H^2$ in MKS units. Thus resistance changes of a part in 10^{12} are possible in our example.

Given
$$\rho(z) = \rho_0(1 + \sigma H^2(z)), \tag{16}$$

which is quite general since a linear contribution does not produce a 3rd order IM of interest anyway, we need to know $H(z)$. Using the same path integral formulation used in the example above, if we take $H(z)_{z\to\infty} = 0$,

$$H(z) = \int_z^\infty J_{IN}(z)dz. \tag{17}$$

For inputs at two frequencies, we can write the primary current, allowing for skin effect,

$$J_{IN}(z) = \mathrm{Re}\left\{(e^{i\omega_1 t} + \beta e^{i\omega_2 t})\,\frac{E_1}{\rho_0}\,e^{\frac{-(1+i)z}{\delta_0}}\right\}. \tag{18}$$

This then leads to $H(z)$ and to $\rho(z)$ through (16),

$$\rho(z) = \rho_0\left[1 + \frac{\sigma\delta_0^2 E_1^2}{4\rho_0^2}\,e^{-\frac{2z}{\delta_0}}\left\{(1+\beta^2) + \sin\left[2\omega_1 t - \frac{2z}{\delta_0}\right] + 2\beta\cos(\omega_1 - \omega_2)t\right.\right.$$
$$\left.\left. + 2\beta\sin\left[[\omega_1 + \omega_2]t - \frac{2z}{\delta_0}\right] + \beta^2\sin\left[2\omega_2 t - \frac{2z}{\delta_0}\right]\right\}\right]. \tag{19}$$

If Eq. (19) is combined with initial input signals, various frequency terms again result. The only parts of $\rho(z)$ which lead to IM products at $(2\omega_1 - \omega_2)$ are

$$\rho(z) = \frac{\sigma E_1^2\delta_0^2}{4\rho_0}\,e^{-\frac{2z}{\delta_0}}\left[\sin\left[\omega_1 t - \frac{2z}{\delta_0}\right] + 2\beta\cos(\omega_1 - \omega_2)t\right]. \tag{20}$$

Unlike the resistive heating case, here both terms in the brackets remain significant. When (20) is combined with the input signals, we obtain a form similar to Eq. (6)

$$J(z) = \mathrm{Re}\left\{\frac{E_1}{\rho_0}(e^{i\omega_1 t} + \beta e^{i\omega_2 t})\left[1 - \frac{\rho(0) - \rho_0}{\rho_0}\right]\left[1 + \frac{(1+i)}{2\delta_0}\int_0^z \frac{\rho(z) - \rho_0}{\rho_0}dz\right]e^{-\frac{(1+i)z}{\delta_0}}\right\}. \tag{21}$$

The results analogous to Eqs. (7), (9), (10) and (11) then become

$$J_{IM}(z) = -\frac{\beta\sigma E_1^3 \delta_0^2 e^{-\frac{z}{\delta_0}}}{16\rho_0^3}\left\{\left[\left(\frac{7}{2}+\frac{1}{2}e^{-\frac{2z}{\delta_0}}\right)\cos\frac{z}{\delta_0}+\left(1+e^{-\frac{2z}{\delta_0}}\right)\sin\frac{z}{\delta_0}\right]\cos(2\omega_1-\omega_2)t\right.$$

$$\left.+\left[\left(\frac{5}{2}+\frac{1}{2}e^{-\frac{2z}{\delta_0}}\right)\sin\frac{z}{\delta_0}+\left(3-e^{-\frac{2z}{\delta_0}}\right)\cos\frac{z}{\delta_0}\right]\sin(2\omega_1-\omega_2)t\right\}, \tag{22}$$

$$J_{IM} = \frac{-5\beta\sigma E_1^3\delta_0^3}{32\rho_0^3}\left[\cos(2\omega_1-\omega_2)t+\sin(2\omega_1-\omega_2)t\right], \tag{23}$$

and thus

$$\overline{J_{IM}^2}R_S = \frac{25}{16}\beta^2\left(\frac{\sigma^3\delta_0^2}{\rho_0^2}\right)\left(\frac{\delta_0 E_1^2}{4\rho_0}\right)^3 = \frac{25}{16}\,\frac{\beta^2}{(1+\beta^2)^3}\left(\frac{\sigma^2\delta_0^2}{\rho_0^2}\right)\bar{p}_{IN}^3 = \bar{p}_{IM}. \tag{24}$$

Comparing this with Eq. (11) and (13) we find that for equal input power and input losses

$$\frac{P_{IML}\text{ (magnetoresistive)}}{P_{IML}\text{ (thermoresistive)}} = \frac{\bar{p}_{IM}\text{ (magnetoresistive)}}{\bar{p}_{IM}\text{ (thermoresistive)}} = \frac{25}{16}\left(\frac{\sigma^2}{\rho_0^2}\right)\left(\frac{G^2T_0^2}{f(x)}\right). \tag{25}$$

Thus the same functional behavior exists as for Eq. (14), except that the result is not sensitive to reasonable changes in the sideband position, signified by the absence of the x dependence in (24). Numerically for copper the ratio (25) takes the value $1.34 \times 10^{-7}/f(x)$ and for the two cases considered in Part 1. with $f(x) = .0028$, or $f(x) = .031$, the magnetoresistive IM power is found to be down from the thermoresistive values by 43 dB and 54 dB, respectively. For non-magnetic materials consideration of reasonable values of $f(x)$ indicates that the magnetoresistive effects are insignificant.

3. DIRECT VARIATION IN RESISTIVITY WITH CURRENT

If $\rho(z) = \rho_0(1+\xi J_{IN}(z)^2)$ where $J_{IN}(z)$ represents the primary current density. then the appropriate part of $J_{IN}(z)^2$ leading to the 3rd order intermodulation signal can be extracted from (1) as

$$J_{IN}(z)^2 = \frac{E_1^2}{2\rho_0^2}e^{-\frac{2z}{\delta_0}}\left[\cos\left(2\omega_1 t-\frac{2z}{\delta_0}\right)+2\beta\cos(\omega_1-\omega_2)t\right]. \tag{26}$$

Then writing

$$\frac{1}{J(z)}\frac{dJ(z)}{dz} = -\frac{(1+i)}{\delta_0(\rho/\rho_0)^{1/2}} = -\frac{(1+i)}{\delta_0(1+\xi J_{IN}(z)^2)^{1/2}} \approx -\frac{(1+i)}{\delta_0}(1-\xi J_{IN}(z)^2/2), \tag{27}$$

we obtain

$$J(z) = \mathrm{Re}\left\{J(0)\exp\left[-\frac{(1+i)z}{\delta_0}\right]\left[1+\frac{\xi(1+i)}{2\delta_0}\int_0^z J_{IN}(z)^2dz\right]\right\}. \tag{28}$$

With $J(0) = \dfrac{E_1}{\rho_0}(e^{i\omega_1 t}+\beta e^{i\omega_2 t})(1-\xi J_{IN}(0)^2)$, to lowest order one gets

$$J_{IM}(z) = \frac{\xi\beta E_1^3 e^{-\frac{z}{\delta_0}}}{2\rho_0^3}\left\{\left[-\left(1+\frac{1}{2}e^{-\frac{2z}{\delta_0}}\right)\cos\frac{z}{\delta_0} + \left(\frac{5}{8}-\frac{3}{8}e^{-\frac{2z}{\delta_0}}\right)\sin\frac{z}{\delta_0}\right]\cos(2\omega_1-\omega_2)t\right.$$

$$\left. - \left[\left(\frac{3}{8}-\frac{3}{8}e^{-\frac{2z}{\delta_0}}\right)\cos\frac{z}{\delta_0} + \frac{1}{2}e^{\frac{-2z}{\delta_0}}\sin\frac{z}{\delta_0}\right]\sin(2\omega_1-\omega_2)t\right\}. \tag{29}$$

Integrated over z, then this leads to

$$J_{IM} = -\frac{\beta\xi E_1^3\delta_0}{16\rho_0^3}\left[3\cos(2\omega_1-\omega_2)t + \sin(2\omega_1-\omega_2)t\right]. \tag{30}$$

Thus

$$\overline{J_{IM}^2}R_S = \frac{\rho_0}{\delta_0}\left(\frac{\beta E_1^3}{\rho_0^3}\right)^2\frac{5\xi^2\delta_0^2}{256} = \frac{5}{4}\frac{\beta^2\xi^2}{\delta_0^2\rho_0^2}\left(\frac{\delta_0 E_1^2}{4\rho_0}\right)^3 = \frac{5}{4}\frac{\beta^2}{(1+\beta^2)^3}\left(\frac{\xi^2}{\delta_0^2\rho_0^2}\right)\bar{p}_{IN}^3 \tag{31}$$

which can be compared with (11) or (24).

This effect is hard to evaluate numerically. Any experimental determination of the coefficient ξ would have to be separated from the magnetoresistive effect dissussed in Part 2. Although the influence of skin depth is different in (23) and (31), one might anticipate IM contributions of similar magnitudes in the two cases.

4. RESISTIVE HEATING AND MAGNETORESISTIVE IM IN FERROMAGNETIC COMPONENTS

When components are composed of ferromagnetic materials, the kinds of components chosen in the previous numerical cases become so lossy as to be impossible for consideration. For a more suitable component we chose a 10 cm long coaxial line similar to RG-19 with $Z_0 = 50\Omega$, ID = .435 cm, OD = 1 cm. For such a line, standard expressions give attenuation = $8.68\alpha_c$ dB/m, where $\alpha_c = 1.05$ R_s (cf. Eq. (8)). The frequencies will be chosen as $\nu_1 = 270$ MHz, $\nu_2 = 245$ MHz. Thus with copper elements in this line there will be a loss of .00376 dB or 9×10^{-4} of input power. If nickel components instead are used, the corresponding loss will change with skin depth depending on the proper value of the permeability μ. This will be taken as the average small signal value of $\frac{dB}{dH}$ and will be presumed constant here (but see Part 5.). If μ is assumed to be 100 μ_0 or 500 μ_0 the fractional loss of input power becomes .019 or .043, respectively. Further assuming $\beta = 1$ and a total input power of 60W, we can then evaluate the IM products for copper or for nickel elements. In this part and in Part 5., the permeabilities used are those suitable to low frequencies. They are still roughly appropriate in UHF systems, but may decrease by an order of magnitude on going to microwave frequencies.[4]

The coaxial component will be treated in an average fashion, with equal power densities on both conductors. One can alternatively distinguish the two, assuming equal total currents on both. In this latter case, the current densities and the total powers absorbed vary inversely as the radii. This implies that the inner conductor is by far the more significant IM source. But since the ratios are known, one can relate the separated conductor results to the average case with the same total absorption. We find for $q = \frac{r_2}{r_1}$ that $\bar{p}_{IM}$ from the outer conductor alone is $1/q^2(1+q)^2$ times the average calculated, while that from the inner conductor is $q^4/(1+q)^2$ times the average. With the coax dimensions chosen, this means that the resultant P_{IML} from the inner conductor is 4 dB above that found from the average, while the outer conductor gives a result which is down 18 dB and negligible in comparison.

If we evaluate P_{IML} from Eq. (14), averaged as above, we find for copper, since $\bar{p}_{IN} = 12\ \mathrm{W/m^2}$ and $P_{IN\ \mathrm{lost}} = 0.054$ W, that $P_{IML} = .62 \times 10^{-27}$W or only -242 dBm due to resistive heating. Magnetoresistive effects are still 43 dB below this, since the frequencies and thus $f(x)$ were chosen as for the first example in Part 1.

But going now to nickel elements we must alter the material parameters: $\rho_0(Ni) = 4.5\rho_0(Cu)$, $G(Ni) = \frac{1}{6.5}G(Cu)$, $C_h(Ni) = 1.15C_h(Cu)$, and we will look at both $\mu = 500\mu_0$ and $\mu = 100\mu_0$. We then obtain

$\mu = 500\mu_0$	$\mu = 100\mu_0$
$P_{IN\ lost} = 2.58$ W	$P_{IN\ \mathrm{lost}} = 1.14$ W
$R_s = 47.5\ R_s(Cu)$	$R_s = 21\ R_s(Cu)$
$\delta_0 = \frac{1}{10.5}\ \delta_0(Cu)$	$\delta_0 = \frac{1}{4.7}\ \delta_0(Cu)$
$x = 0.26x(Cu)$	$x = 0.58x(Cu)$
$f(x) = 0.136$	$f(x) = 0.0153$
$P_{IML} = 0.59 \times 10^{-19}$ W *or* -162 dBm	$P_{IML} = 0.25 \times 10^{-21}$ W *or* -186 dBm

The IM levels here are for resistive heating effects. Although they are still at low levels, they are considerably stronger than the corresponding copper value of -242 dBm.

To evaluate the magnetoresistive contribution, we need a value for the coefficient σ in Eq. (16). Material is available in a discussion[5] by Jan which shows that below about half of saturation magnetization, $\left(\frac{M_s}{2}\right)$, both transverse and longitudinal fields give $\frac{\Delta\rho}{\rho_0}$ proportional to M^2. Numerically, Jan's curves for nickel in transverse fields show $\frac{\Delta\rho}{\rho_0} = -4 \times 10^{-2}\left(\frac{M}{M_s}\right)^2$. Generalizing this somewhat, if a strong transverse field H_0 is applied, an expansion about H_0 gives

$$\frac{\Delta\rho(H) - \Delta\rho(H_0)}{\rho_0} = b\left\{2M(H_0)\frac{dM}{dH}(H_0)\ (H - H_0) + \left[\left(\frac{dM}{dH}(H_0)\right)^2 + M(H_0)\frac{d^2M}{dH^2}(H_0)\right](H - H_0)^2\right\} \tag{32}$$

where $b = -\frac{4 \times 10^{-2}}{M_s^2}$. Only the quadratic term will contribute to the 3rd order IM. (For M near M_s, $\frac{\Delta\rho}{\rho_0}$ is experimentally quite linear in H, so saturation must drastically reduce IM production.) Within the coefficient of the quadratic term, the first part is positive; the second will be positive and augment the first at the lower end of the magnetization curve, but must eventually become negative and weaken the first. To simplify, we will take the first coefficient alone, giving $\sigma = b\left(\frac{dM}{dH}(H_0)\right)^2 = b\left(\frac{\mu}{\mu_0} - 1\right)^2$. Since $M_s = 4.85 \times 10^5$A/m we obtain

	$\mu = 500\mu_0$	$\mu = 100\mu_0$
	$\sigma = -4.24 \times 10^{-8}\ (\mathrm{A/m})^{-2}$	$= -1.69 \times 10^{-9}\ (\mathrm{A/m})^{-2}$
$\frac{P_{IML}\ \text{(magnetoresistive)}}{P_{IML}\ \text{(thermoresistive)}}$	$= 1.3 \times 10^9$ or $+91$ dB	$= 1.8 \times 10^7$ or $+72$ dB
Magnetoresistive P_{IML}	$= -71$ dBm	$= -114$ dBm

Thus for magnetic materials, magnetoresistive effects are extremely strong sources of IM signals. Despite this, however, we find that an even more important source exists in the direct variation of the permeability with current. This is discussed in the next part.

It should be emphasized that these results are only approximate. The value of μ is a continuously varying function of field strength and frequency and the skin depth is thus itself a function of depth. Also, especially with external fields applied, there are directional variations in effective permeability since the ability of a small field to change the magnitude of magnetization is much greater for fields parallel rather than perpendicular to existing magnetization. For example, a solenoidal field parallel to the coaxial component is parallel to the currents in the conductors and normal to the fields which they produce, and so should decrease the effective permeability and consequent IM production more effectively than a transverse applied field. Furthermore, shape effects on the demagnetizing field make the solenoidal field much more effective in saturating the magnetic components, requiring only about $10^{-2}T$ as opposed to several tenths Tesla for a transverse applied field.

5. IM PRODUCTION DUE TO VARIATIONS IN PERMEABILITY IN FERROMAGNETIC COMPONENTS

To illustrate the significance of this effect, ignoring other contributions including imaginary μ'', we allow the permeability to vary in the form

$$\mu(z) = \mu_u \, (1 + DH(z)^2) \tag{33}$$

since linear variations will not lead to 3rd order IM's. We are interested here in fields produced by the currents in the conductors. From the primary currents, the parts of $H^2(z)$ which contribute to the 3rd order IM become (cf Eqs. (16) and (20))

$$H^2(z) = \frac{E_1^2\sigma_0^2}{4\rho_0^2} e^{-\frac{2z}{\delta_0}} \left[\sin\left\{2\omega_1 t - \frac{2z}{\delta_0}\right\} + 2\beta \cos(\omega_1 - \omega_2)t\right]. \tag{34}$$

This is to be incorporated into an expression analogous to Eq. (5)

$$\frac{1}{J(z)}\frac{dJ(z)}{dz} = \frac{(1+i)}{\delta_0(\mu_u/\mu(z))^{1/2}} - \frac{(1+i)}{\delta_0}\left\{1 + \frac{DH^2(z)}{2}\right\} \tag{35}$$

for which the solution, similar to (21), becomes

$$J(z) = \mathrm{Re}\left\{\frac{E_1}{\rho_0}(e^{i\omega_1 t} + \beta e^{i\omega_2 t})\left[1 - \frac{(1+i)D}{2\delta_0}\int_0^z H^2(z)dz\right] e^{-\frac{(1+i)z}{\delta_0}}\right\}. \tag{36}$$

From this we obtain

$$J_{IM}(z) = \frac{\beta D\delta_0^2 E_1^3}{32\rho_0^3} e^{-\frac{z}{\delta_0}} \left\{\left(e^{\frac{-2z}{\delta_0}} - 1\right)\left(\cos\frac{z}{\delta_0} + 2\sin\frac{z}{\delta_0}\right)\cos(2\omega_1 - \omega_2)t \right. \tag{37}$$

$$\left. + \left[(e^{-\frac{2z}{\delta_0}} - 1)(\sin\frac{z}{\delta_0} - 2\cos\frac{z}{\delta_0}) - 2\sin\frac{z}{\delta_0}\right]\sin(2\omega_1 - \omega_2)t\right\}.$$

The integrated form of (37) gives

$$J_{IM} = -\frac{\beta D E_1^3\delta_0^2}{32\rho_0^3}\,[\cos(2\omega_1 - \omega_2)t + \sin(2\omega_1 - \omega_2)t], \tag{38}$$

in close correspondence with Eq. (23), as we would expect from the similarities of (16) and (33). (One point of difference arises from the fact that at the surface ($z = 0$) the present model introduces no IM current whereas magnetoresistivity does.) From Eqs. (37) and (23) we can thus write

$$\frac{J_{IM}\ (\text{variable } \mu)}{J_{IM}\ (\text{magnetoresistive})} = \frac{D}{5\sigma}, \quad \frac{P_{IML}\ (\text{variable } \mu)}{P_{IML}\ (\text{magnetoresistive})} = \left(\frac{D}{5\sigma}\right)^2. \tag{39}$$

In this formulation, Eq. (33) shows that $D = \dfrac{d^2\mu}{\mu_u dH^2}$ where μ_u represents the average value of permeability in the average static field present. Typically we expect μ to be an increasing function of H for small fields and to be decreasing in fields above a few hundredths Tesla. This suggests that as applied fields increase μ will pass through regions of constant slope, which should produce minima in IM power levels. The ratio in (39), using (32), becomes

$$\frac{D}{5\alpha} = \frac{d^2\mu}{dH^2} \Bigg/ 5b\mu_u\left(\left(\frac{dM}{dH}\right)^2 + M\frac{d^2M}{dH^2}\right). \tag{40}$$

Using $M = \left(\dfrac{\mu}{\mu_0} - 1\right)H$, (40) can be written as

$$\frac{D}{5\alpha} = \frac{1}{5b\mu_u}\frac{d^2\mu}{dH^2}\left[\left(\frac{\mu}{\mu_0} - 1\right)^2 + 4\left(\frac{\mu}{\mu_0} - 1\right)\frac{H^2}{\mu_0^2}\left(\frac{d\mu}{dH}\right)^2 + \left(\frac{\mu}{\mu_0} - 1\right)\frac{H^2}{\mu_0}\frac{d^2\mu}{dH^2}\right]^{-1}. \tag{41}$$

If we are concerned mainly with small external applied fields, curves for the magnetization of iron[6] can be used to estimate these values. (Nickel and iron values are similar enough to give comparable results.) As in Part 4., we use low frequency permeability data. At microwave frequencies we can anticipate an order of magnitude decrease μ.[4] We also assume that the b value of $-04/\mathrm{M}^2$ is approximately correct. For iron, at $H = 80$ A/m (1 Oe), $\mu_u \approx 3000\,\mu_0$, $\dfrac{d\mu}{dH} \approx 125\,\mu_0$, $\dfrac{d^2\mu}{dH^2} \approx 11\,\mu_0$, and $M_s \approx 1.6 \times 10^6$A/m. Hence $\dfrac{D}{5\sigma}$ becomes about 106 which then implies a power ratio of about 40 dB. This factor is difficult to establish accurately, and is very subject to material and environmental influences. Nonetheless, this contribution to IM signals is obviously so extremely significant that it reinforces the argument that ferromagnetic materials must be entirely excluded from high sensitivity multiplex circuits.

6. INTERMODULATION DUE TO NON-LINEAR DIELECTRICS

If dielectrics are present in a system, any variation in dielectric properties with applied field will serve to modulate incoming signal voltages in a manner analogous to the modulation of currents in Part 1. This effect has been addressed in both the Philco[1] and TRW[2] studies cited, and again here the aim is to rectify some apparent discrepancies. The approach below is similar in general outline to that of the TRW study. A solution has also been obtained in the manner of Part 1.; it yields results substantially equivalent to those below when the largely reactive nature of the IM source impedance is taken into account.

Variations in dielectric properties may occur in direct response to electric fields, through heating, or by electrostriction. The general form of the solution is applicable to a variety of dielectric media, such as molecular absorbers or ionizing gases. In each case, of course, the nature of the response to applied fields must be determined.

We start by writing equations for a generalized transmission line in the form

$$\frac{\partial V}{\partial y} = -L\frac{\partial I}{\partial t} - RI \text{ and } \frac{\partial I}{\partial y} = -C_0\epsilon\frac{\partial V}{\partial t} \tag{42}$$

where L, R, and C_0 (vacuum value) are the inductance, resistance, and capacity per unit length of line. The conductance of the dielectric will be included in a complex relative dielectric constant ϵ. We now take

$$\epsilon = \epsilon_1(1 + \alpha V^2) - i\epsilon_2(1 + \eta V^2) \tag{43}$$

both because quadratic terms are needed for IM production and because the dielectric response is expected to be insensitive to the sign of the applied field. The relationship between the factors α and η and corresponding factors suitable for multiplying E^2 will be discussed later. Substitution of (43) into Eq. (42) leads to

$$\frac{\partial^2 V}{\partial y^2} = -L\frac{\partial^2 I}{\partial t \partial y} - R\frac{\partial I}{\partial y} = LC_0\frac{\partial^2 V}{\partial t^2}(\epsilon_1 - i\epsilon_2) + LC_0(\alpha\epsilon_1 - i\eta\epsilon_2)\frac{\partial}{\partial t}\left[V^2\frac{\partial V}{\partial t}\right] \tag{44}$$
$$+ RC_0(\epsilon_1 - i\epsilon_2)\frac{\partial V}{\partial t} + RC_0(\alpha\epsilon_1 - i\eta\epsilon_2)V^2\frac{\partial V}{\partial t}.$$

We assume that we can write $V = V_0 + V_1 + \ldots$ in descending order of magnitude based on the smallness of α and η. Equations for V_0 and V_1 are then given by

$$\frac{\partial^2 V_0}{\partial y^2} - LC_0(\epsilon_1 - i\epsilon_2)\frac{\partial^2 V_0}{\partial t^2} - RC_0(\epsilon_1 - i\epsilon_2)\frac{\partial V_0}{\partial t} = 0 \tag{45}$$

and

$$\frac{\partial^2 V_1}{\partial y^2} - LC_0(\epsilon_1 - i\epsilon_2)\frac{\partial^2 V_1}{\partial t^2} - RC_0(\epsilon_1 - i\epsilon_2)\frac{\partial V_1}{\partial t} = LC_0(\alpha\epsilon_1 - i\eta\epsilon_2)\frac{\partial}{\partial t}\left[V_0^2\frac{\partial V_0}{\partial t}\right]$$
$$+ RC_0(\alpha\epsilon_1 - i\eta\epsilon_2)V_0^2\left[\frac{\partial V_0}{\partial t}\right]. \tag{46}$$

The lowest order solution for one signal propagating to positive y can be written in the form

$$V_0 = U \operatorname{Re} e^{i\omega t}e^{-i\tau y} = U \operatorname{Re} e^{i(\omega t - \kappa y)}e^{-\gamma y}. \tag{47}$$

Here $\tau^2 \equiv \omega C_0(\omega L - iR)(\epsilon_1 - i\epsilon_2) \equiv (\kappa - i\gamma)^2$.

We now assume that there are two primary input signals, so that

$$V_0 = U \operatorname{Re}\,[e^{i(\omega_1 t - \kappa_1 y)}e^{-\gamma_1 y} + \beta e^{i(\omega_2 t - \kappa_2 y)}e^{-\gamma_2 y}]. \tag{48}$$

Then Eq. (46) will contain on the right side terms at several different frequencies. We are interested in the IM at $\omega_{IM} = 2\omega_1 - \omega_2$. After evaluating $V_0^2\dfrac{\partial V_0}{\partial t}$ we find that we can rewrite Eq. (46) as

$$\frac{\partial^2 V_1}{\partial y^2} - LC_0(\epsilon_1 - i\epsilon_2)\frac{\partial^2 V_i}{\partial t^2} - RC_0(\epsilon_1 - i\epsilon_2)\frac{\partial V_1}{\partial t} = \tag{49}$$
$$-\frac{1}{4}\beta U^3(LC_0\omega^2 - iRC_0\omega)(\alpha\epsilon_1 - i\eta\epsilon_2)e^{i(\omega t - \kappa y)}e^{-\gamma y}$$

where we want to keep the real part of V_1. Here $\omega \equiv \omega_{IM} \equiv 2\omega_1 - \omega_2$, $\kappa \equiv 2\kappa_1 - \kappa_2$, and $\gamma \equiv 2\gamma_1 + \gamma_2$. Note that κ and γ are not in general the propagation constants for a signal at ω_{IM}, although to lowest order κ will be. These quantities will be discussed again later.

The solution of Eq. (49) which satisfies the conditions that $V_1 = 0$ both at $y = 0$ and as $y \to \infty$ is, to first order in α and η,

$$V_1 = \frac{\beta U^3\omega C_0}{4}\operatorname{Re} e^{i\omega t}\frac{\{(\omega L - iR)(\alpha\epsilon_1 - i\eta\epsilon_2)\}\{e^{-[\omega C_0(\omega L - iR)(\epsilon_1 - i\epsilon_2)]^{1/2}y} - e^{-i(\kappa - i\gamma)y}\}}{\{(\gamma + i\kappa)^2 + \omega C_0(\omega L - iR)(\epsilon_1 - i\epsilon_2)\}} \tag{50}$$

If the expression for τ^2 (Eq. 47) when $\omega = \omega_{IM}$ is used to define new quantities κ_{IM} and γ_{IM} we can then write

$$V_{IM} = \frac{\beta U^3}{4} \operatorname{Re} e^{i\omega_{IM}t} \frac{\{\tau_{IM}^2(\alpha\epsilon_1 - i\eta\epsilon_2)\}\{e^{-i\kappa_{IM}y}e^{-\gamma_{IM}y} - e^{-i(2\kappa_1-\kappa_2)y}e^{-(2\gamma_1+\gamma_2)y}\}}{\{\tau_{IM}^2 - (2\kappa_1 - \kappa_2 - 2i\gamma_1 - i\gamma_2)^2\}(\epsilon_1 - i\epsilon_2)} \tag{51}$$

Evaluated at the full length Y of the line, this then lead to

$$P_{IML} = \overline{V_{IM}^2}(Y)/Z_0. \tag{52}$$

The characteristic impedance Z_0 will also be frequency dependent, so that ideal matching cannot occur at all frequencies if $\frac{R}{\omega L}$ is significant. It is to be expected, however, that both $\frac{R}{\omega L}$ and $\frac{\epsilon_2}{\epsilon_1}$ will be small for real cases of interest in which case Z_0 can be taken to sufficient accuracy as $\left(\frac{L}{C_0\epsilon_1}\right)^{1/2}$.

Two cases of special interest are the general case when $\omega_{IM} \approx \omega_1 \approx \omega_2$, and the case where $\frac{\epsilon_2}{\epsilon_1}$ and $\frac{R}{\omega L}$ are both small, but frequencies are general.

When the IM frequency is very close to both carriers, we can approximate $\omega_{IM} = \omega_1 = \omega_2$, $\kappa_{IM} = \kappa_1 = \kappa_2$, and $\gamma_{IM} = \gamma_1 = \gamma_2$. From Eq. (51) we then find

$$V_{IM} = \frac{\beta U^3}{16} \operatorname{Re} e^{i(\omega_1 t - \kappa_1 y)} \frac{\{(\kappa_1 - i\gamma_1)^2(\alpha\epsilon_1 - i\eta\epsilon_2)\}}{\{(2\gamma_1^2 + i\kappa_1\gamma 1)(\epsilon_1 - i\epsilon_2)\}}(e^{-\gamma_1 y} - e^{-3\gamma_1 y}) \tag{53}$$

The y dependence of the amplitude, all contained in the last parentheses, leads to a saturation effect as described by TRW, with a maximum at $e^{-2\gamma_1 y} = \frac{1}{3}$, i.e., where P_{IN} is down 4.8 dB. This will generally involve transmission lines significantly longer than 3 m so that the effect would usually not be observed. A similar saturation effect will occur also in other IM processes.

In the case where $\frac{\epsilon_2}{\epsilon_1}$ and $\frac{R}{\omega L}$ are both small, the quantities κ and γ can be written

$$\kappa \approx \omega\,(LC_0\epsilon_1)^{\frac{1}{2}} \text{ and } \gamma \approx \frac{1}{2}\kappa\left(\frac{\epsilon_2}{\epsilon_1} + \frac{R}{\omega L}\right). \tag{54}$$

Hence we can take $\kappa_{IM} = 2\kappa_1 - \kappa_2$ and $\gamma_{IM} = 2\gamma_1 - \gamma_2$. Taken to lowest order, Eq. (51) leads to

$$V_{IM} \approx \frac{\beta U^3}{8}\kappa_{IM}y\left\{\alpha\sin(\omega_{IM}t - \kappa_{IM}y) + \left(\frac{\epsilon_2}{\epsilon_1}\right)(\alpha - \eta)\cos(\omega_{IM}t - \kappa_{IM}y)\right\}. \tag{55}$$

Note that to this approximation, the exponential factors have vanished from the result. If, in fact, $\epsilon_2 = R = 0$, so that only the real part of ϵ is changing, Eq. (46) can be solved directly to obtain the first term here. It can be seen that generally α will dominate η as an IM contributor. Only if α is essentially zero, perhaps as in an absorbing gas, will the η contribution be significant.

For the two special cases above, we can readily write the IM power at the load, taking $Z_0 = \left(\frac{L}{C_0\epsilon_1}\right)^{\frac{1}{2}}$. From Eq. (53) we obtain

$$P_{IML} = \frac{\beta^2 U^6}{512 Z_0}\frac{(\kappa_1^2 + \gamma_1^2)^2}{\gamma_1^2(\kappa_1^2 + 4\gamma_1^2)}\frac{(\alpha^2\epsilon_1^2 + \eta^2\epsilon_2^2)}{(\epsilon_1^2 + \epsilon_2^2)}e^{-2\gamma_1 Y}(1 - e^{-2\gamma_1 Y})^2. \tag{56}$$

From Eq. (55), the result for small ϵ_2 and R becomes

$$P_{IML} = \frac{\beta^2 U^6}{128 Z_0} \kappa_{IM}^2 \alpha^2 Y^2 . \tag{57}$$

In the absence of α, α^2 in Eq. (57) or (58) below is to be replaced by $\eta^2 \left(\frac{\epsilon_2}{\epsilon_1}\right)^2$. In terms of the input powers $P_1 = \frac{U^2}{2Z_0}$ and $P_2 = \frac{\beta^2 U^2}{2Z_0}$, the quantity $(\beta U^3)^2 = 8Z_0^3 P_1^2 P_2$, while the propagation constant κ_{IM} in Eq. (57) can be taken as $2\pi/\lambda_{IM}$. Hence Eq. (57) can also be written

$$P_{IML} = \frac{1}{4}\pi^2 Z_0^2 P_1^2 P_2 \, \alpha^2 \left(\frac{Y}{\lambda_{IM}}\right)^2 . \tag{58}$$

Here λ_{IM} is the IM wavelength in the transmission line. If Y^2 is large enough it should be replaced in the next order of approximation by $e^{-2\gamma_1 Y}\,(1-e^{-2\gamma_1 Y})^2/4\gamma_1^2$. Then the maximum P_{IML}, when $e^{-2\gamma_1 Y} = \frac{1}{3}$, is given by Eq. (58) if Y^2 is replaced by $1/27\gamma_1^2$. On the other hand, a discrete dielectric element can also be treated using Eq. (58) if Y is taken to be the length of that element.

In order to connect the parameter α with the more significant coefficient α' which expresses dependence on electric field, we can treat the transmission line structure as a capacitor with ϵ dependent on E^2 and then find to lowest order the corresponding connection between capacity and V^2. For example, if the volume occupied by the dielectric does not change, a structure with plane parallel plates results simply in $\alpha = \frac{\alpha'}{d^2}$, where d is the spacing of the plates. In the case of coaxial cylindrical plates, the relationship becomes $\alpha = \frac{1}{2}\alpha' \, (r_1^{-2} - r_2^{-2}) \left(\ln\frac{r_2}{r_1}\right)^{-3}$. The same relationships, if needed, will connect η and η'.

As one example, we now apply Eq. (58) to a case considered in the Philco study: molecular absorption by 1% water vapor in air, all contained in an x-band waveguide. We use the same conditions given in the Philco study, $P_1 = 6$ kW, $P_2 = 60$ W, $Z_0 = 448\ \Omega$, $Y = 10$ m, $\lambda_{IM} = 0.0445$ m, $d = .01$ m, $\epsilon_1 = 1$, $\alpha \approx 0$, dielectric attenuation 2.9×10^{-6} dB/m, $\eta' E^2 = 2 \times 10^{-4}$ for $E = 1.4 \times 10^5$ V/m. Treating the waveguide as a parallel plate capacitor leads to $\eta' = 10^{-14}$, $\epsilon_2 = 4.7 \times 10^{-9}$, and to $P_{IML} = 1.2 \times 10^{-17}$ W or -139 dBm. This value is below the -129 dBm estimated in the Philco study, but is not negligible. With greater power on either input, higher humidity, a longer waveguide, or the 15 to 20 dB increase estimated by Philco for more refined analysis of the molecular absorption process, water vapor becomes a significant source of IM signals. (In comparison, the same waveguide, as treated in Part 1, with 33 dB added for increased power and length contributes only -157 dBm through resistive heating.) This water vapor effect is a result of the molecular rotational absorptions at 22.2 GHz and will become more pronounced as that frequency is approached. Below x-band frequencies, this is the only absorptions process of interest in normal air, but at higher frequencies both water vapor and molecular oxygen absorption (at 60 GHz) will result in strong IM generation as well as significant signal propagation losses.

When a solid dielectric is considered, the mass rather than volume of dielectric is conserved. Allowance for the change dielectric volume modifies the relationships between α and α' above. These will then be given by $\alpha = \alpha' \left(\frac{\epsilon - 1}{d^2}\right)$ or $\alpha = \frac{1}{2}\alpha' \left(r_1^{-2} - r_2^{-2} - \frac{2\epsilon}{r_2^2} \ln \frac{r_2}{r_1}\right) \left(\ln\frac{r_2}{r_1}\right)^{-3}$, respectively, on the assumption that $\frac{\Delta\chi}{\chi} = \frac{\Delta m}{m}$ (χ, susceptibility; m, density), as in electrostriction.

Electrostriction is expected to be the principal source of non-linearity in good non-polar dielectrics.[7] The TRW study considers the effect of electrostriction on teflon in a coaxial line. Using the

conditions of the TRW example for a 300 MHz IM signal, with $\alpha' \approx (0.8$ to $25) \times 10^{-21}$ $(\mathrm{V/m})^{-2}$, $r_2 = 2.4 \times 10^{-3}$ m $= 3r_1$, $\lambda_{IM} = 0.7$ m, $Z_0 = 47\Omega$, $P_1 = P_2 = 30$ W, $\epsilon_1 = 2$, the above relationship for α combined with Eq. (58) leads to $P_{IML} = (.01$ to $10) \times 10^{-21} Y^2$ W. A line 1 m long will thus yield P_{IML} of -170 to −200 dBm, probably below detectability.

The value of α' is obtained from the expression

$$\Delta\epsilon = \frac{\epsilon_0 E^2}{K} (\epsilon - 1)^2 \tag{59}$$

where K is the bulk modulus of compressibility. Equation (59) is given by Böttcher[7] based on thermodynamic arguments. The spread in values reflects the range of K values quoted by the TRW report.

Since K is about as low as possible for teflon, the effect of electrostriction is unlikely to be significant with other well chosen dielectrics. If materials with resonance absorption losses, polar properties (permament dipole moments) or anisotropic polarizabilities were used, however, the IM products could be significantly increased, so such materials should be avoided.

Functionally, Eq. (58) and the relationships between α and α' show that P_{IML} will increase quadratically with the number of wavelengths in the transmission line length, and except as Z_0 is altered by dimensional changes will vary inversely with the fourth power of the transverse structural dimensions. Since structure size and power levels will generally be correlated, the conclusion is that this kind of dielectric IM production will be insignificant in all but extreme cases. (For instance, if in the example above, P_1 and P_2 are increased to 1000 W while r_1 and r_2 are trebled, P_{IML} will be increased 27 dB.) The one configuration in which IM production might be significant with good dielectrics is a high-Q resonant structure where the power densities are correspondingly enhanced. Thus it is desirable to exclude all dielectric materials from such elements.

7. SUMMARY

The intermodulation signals contributed by the intrinsic non-linear properties of materials (other than semiconductors) used in multiplex circuits have been calculated. Functional dependences, in terms of parameters accessible externally, have been obtained for the normally dominant 3rd-order IM signals.

Non-magnetic conductors are considered in which resistivity changes either by resistive heating, by associated mangetic fields, or by current density directly. Resistive heating is the most important of these effects.

Ferromagnetic metallic components are treated separately. These materials are entirely unsuited to small signal applications. In addition to causing very high losses, they can generate IM signals 100 dB above thermal noise in realistic circumstances. The danger in the use of such components is illustrated in Chapters II-IV.

Dielectric elements, distributed or not, in which the dielectric properties are functions of electric field, are treated in a generalized fashion. It is found that water vapor in air produces non-negligible IM signals, particularly at microwave frequencies. Dielectric media with no polar properties or resonance losses are generally insignificant as IM sources, although in resonant structures they might lead to observable IM signals.

The results of this chapter lead to the following recommendations concerning materials in multiplex systems:

i.) Ferromagnetic materials (and also semiconductors) should be totally excluded from any part of a system in which multiple signals exist. Leakage effects may require shielding of such materials.

ii.) Water vapor should be excluded from all transmission lines and cavities, particularly for microwave frequencies. Attenuation in the atmosphere is probably the main deterrent to operating above x-band frequencies, but if higher frequencies are used, oxygen also should be removed from the system.

iii.) High power densities should be avoided; since these are inevitable for cavity filters, materials used there (even more than in other parts of the system) must have low resistivity, high heat capacity, high thermal conductivity and low thermal coefficients of resistivity. Surfaces must be clean and free of corrosion products.

iv.) Dielectric materials should be excluded from resonant structures. If used elsewhere they should be non-polar and with minimum loss characteristics.

v.) Paths in which multiple signals pass should be kept to minimum lengths, preferably a meter or less. Mechanical junctions and closures should be kept to a minimum, placed to minimize currents across them, and made to best insure metallic contact.

If the above conditions are satisfied, IM production will probably be dominated either by the junction effects or by resistive heating in resonant elements. In the latter case the example given in Part 1 suggests that for an input power level of 1 kW, 3rd order IM power can be held below about −140 dBm for UHF signals. Because of the functional dependences illustrated in Eq. (15), this level will increase as input power rises or system temperature falls, and will increase as the separation of primary signal frequencies shrinks or as the primary frequencies rise.

SYMBOLS USED

Quantities used only where defined not listed. MKS units throughout. The subscript IM on a quantity means the same quantity as without, but restricted to the IM signal (3rd order, $2\omega_1 - \omega_2$ only).

A	effective conductor surface area
B	magnetic induction
b	defined by Eq. (32)
C_0	capacitance per unit length of transmission line with vacuum dielectric
C_h	heat capacity of conductor
D	coefficient of quadratic magnetic field dependence of permeability
d	spacing of parallel plate capacitor
E	electric field strength
$f(x)$	defined by Eq. (11)
G	thermal conductivity of conductor
H	magnetic field strength; $H(z)$ instantaneous local value
I	instantaneous current in transmission line
$J(z)$	instantaneous local current density; $J_{IN}(z)$ lowest order value due to input signals
J	instantaneous total current at all depths in conductor per unit width of surface
$\bar{J}$	rms value of J
k	Boltzmann constant
L	transmission line inductance per unit length
M	magnetization; M_s saturation magnetization
$p_{IN}(z)$	instantaneous local density of power dissipated, lowest order due to input signals
$\bar{p}_{IN}$	rms power dissipated at all depths per unit surface area, lowest order due to input signals

$\bar{p}_{IML}$	rms IM power dissipated at load due to unit area of IM source
P_{IN}	total rms input power available to load
$P_{IN\,\mathrm{lost}}$	total rms input power lost in transmission to load
P_{IML}	total rms IM power delivered to load
Q	circuit Q factor
R	resistance of transmission line per unit length
R_s	surface resistivity, cf. Eq. (8)
r_1, r_2	inner, outer radii of coaxial transmission line
T	temperature; T_0 ambient; $T(z)$ local instantaneous value above ambient
U	amplitude of rf voltage across transmission line
V	instantaneous voltage across transmission line; V_0, V_1lowest, next lowest order values for changes in dielectric constants.
W	defined by Eq. (4)
x	defined by Eq. (7)
Y	length of transmission line
y	distance along direction of propagation
Z_0	characteristic impedance or matching load impedance
z	depth below surface of conductor
β	ratio of signal amplitudes for two inputs, signal 2/signal 1.
δ_0	skin depth for small signal levels; $\delta(z)$ instantaneous local value
$\epsilon = (\epsilon_1 - i\epsilon_2)$	*relative* dielectric constant for small signal levels
ξ	coefficient of quadratic current density dependent change in resistivity
λ	wavelength in transmission line
μ	magnetic permeability; μ_u permeability for very weak signals; $\mu(z)$ instantaneous local value
ν	circular frequency
ω	radian frequency
ρ_0	resistivity for small signals; $\rho(z)$ instantaneous local value
σ	coefficient of quadratic magnetic field dependent change in resistivity
ϕ	defined by Eq. (4)
ψ	generalization of T_0^{-1}, coefficient of linear thermal dependent change in resistivity
α', η'	coefficients of quadratic electric field dependent changes in real and imaginary dielectric constants
α, η	α', η' modified by transmission line geometry to give changes in dielectric constants with voltage across transmission line
$\tau = (\kappa - i\gamma)$	complex propagation constant

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